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THE RELATION OF EQUIVALENCE FOR POST FUNCTIONS

ED DuCASSE
GERNOT METZE

UNIVERSITY OF ILLINOIS - URBANA, ILLINOIS

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THE RELATION OF EQUIVALENCE FOR POST FUNCTIONS

by

Ed DuCasse and Gernot Metze

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ABSTRACT

An equivalence relation for one-variable functions defined on the Post chain P(n) has been introduced by Wojcik [8] and by Wojcik and Metze [9]. We present here several results characterizing the equivalence classes of this relation, giving us some insight into its nature and structure.

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I. INTRODUCTION

Wojcik [8] and Wojcik and Metze [9] have introduced an equivalence relation for the Post algebra $G[P(n):x]$, the algebra of all one-variable functions defined on the Post chain $P(n)$, which is isomorphic to the lattice $[P(n)]^n$, the direct product of n n-chains. The usefulness of this equivalence relation lies in its application to finding functions which lead to circuit implementations free of operation hazards [8], a type of temporary false output peculiar to transitions between nonadjacent signal values in a multivalued implementation. The relation of equivalence for the elements of $G[P(n):x]$ partitions this algebra in such a way that each equivalence class contains a function which is free of operation hazards. It can be shown, incidentally, that by selecting this function as class representative, a Boolean function may be represented in terms of operation-hazard-free Post functions [8].

This report investigates some of the properties of these equivalence classes. After giving some basic definitions, several characterizations of the relation of equivalence for Post algebras $[P(n)]^n$ are presented. Among these are a necessary and sufficient condition for two elements of $[P(n)]^n$ to be equivalent, and a characterization of the set of all elements equivalent to a given element in terms of a coset-like class of the lattice.

II. FUNDAMENTAL DEFINITIONS

Before introducing the concept of equivalence for Post functions $g(x)$, we recall the definition of Post algebra.

DEFINITION 1: Let n be an integer such that $n \geq 2$. A Post algebra P is a distributive lattice with zero θ and unit U for which the following conditions hold:

AXIOM 1: There exist n elements $e_0, \ldots, e_{n-1}$ satisfying

(1a) $\theta = e_0 \leq e_1 \leq \ldots \leq e_{n-1} = U$;

(1b) if $y \in P$ and $y \cdot e_1 = \theta$, then $y = \theta$;

(1c) if $y \in P$ and $y + e_{i-1} = e_i$ for some i, then $y = e_i$.

AXIOM 2: For each element $y \in P$, there exist n elements $C_0(y), \ldots, C_{n-1}(y)$ in P satisfying

(2a) $C_i(y) \cdot C_j(y) = \theta$ if $i \neq j$;

(2b) $\sum_{i=0}^{n-1} C_i(y) = U$;

(2c) $y = \sum_{i=0}^{n-1} (e_i \cdot C_i(y))$.

As an example of a Post algebra, consider the 3-chain, as shown in Figure 1. For convenience, the elements have been labeled 0,1,2, where $0 < 1 < 2$. The 3-chain is a distributive lattice with $\theta = 0$ and $U = 2$. The elements $e_0 = 0$, $e_1 = 1$ and $e_2 = 2$ satisfy (1a), (1b), and (1c) of Definition 1.

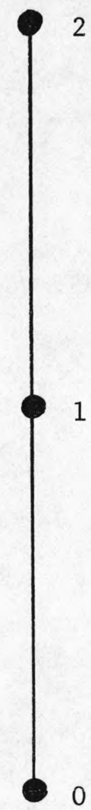

FIGURE 1

THE POST ALGEBRA P(3)

This follows as a special case of a result of DuCasse and Metze [2], or can be computed directly. The functions C_0, C_1, C_2 are also easily identified using a theorem of DuCasse and Metze [2]. The values of these operators for specific arguments are given in Table 1. The $C_i(y)$, $i = 0,1,2$, satisfy (2a) and (2b) of Definition 1 and, together with the elements e_i, $i = 0,1,2$, satisfy Axiom (2c). Thus the 3-chain is a Post algebra, which we denote by P(3). In fact, any n-chain, for $n \geq 2$, can be shown to be a Post algebra [8]. We denote the n-chain, for $n \geq 2$, by P(n).

As a more substantial example, consider the lattice of Figure 2. This poset is the direct product of three 3-chains. Thus its elements can be represented by 3-tuples, each of whose components is either 0, 1 or 2. It is a distributive lattice with $\theta = 000$ and $U = 222$. As was the case with P(3), the elements e_i and functions C_i, $i = 0,1,2$, can be identified from the characterizations given in DuCasse and Metze [2]. The elements $e_0 = 000$, $e_1 = 111$ and $e_2 = 222$ satisfy (1a), (1b) and (1c) of Definition 1. The functions C_0, C_1, and C_2 defined in Table 2 satisfy (2a) and (2b) of Definition 1, and, together with e_0, e_1, and e_2, satisfy Axiom (2c). Thus the lattice of Figure 2 is a Post algebra, which we denote by $[P(3)]^3$. More generally, it has been shown that a direct product of m n-chains, where $m \geq 1$ and $n \geq 2$ are integers, is always a Post algebra; and that every finite Post lattice is such a product [6]. This result will be used frequently in the sequel.

The Post algebra $[P(3)]^3$ and, more generally, the lattices $[P(n)]^n$, will play a prominent role in the remainder of this report; for $[P(n)]^n$ has been shown to be isomorphic to $G[P(n):x]$, the lattice of all

y	$C_0(y)$	$C_1(y)$	$C_2(y)$
0	2	0	0
1	0	2	0
2	0	0	2

TABLE 1

THE FUNCTION VALUES $C_i(y)$, $i = 0,1,2$, FOR $P(3)$

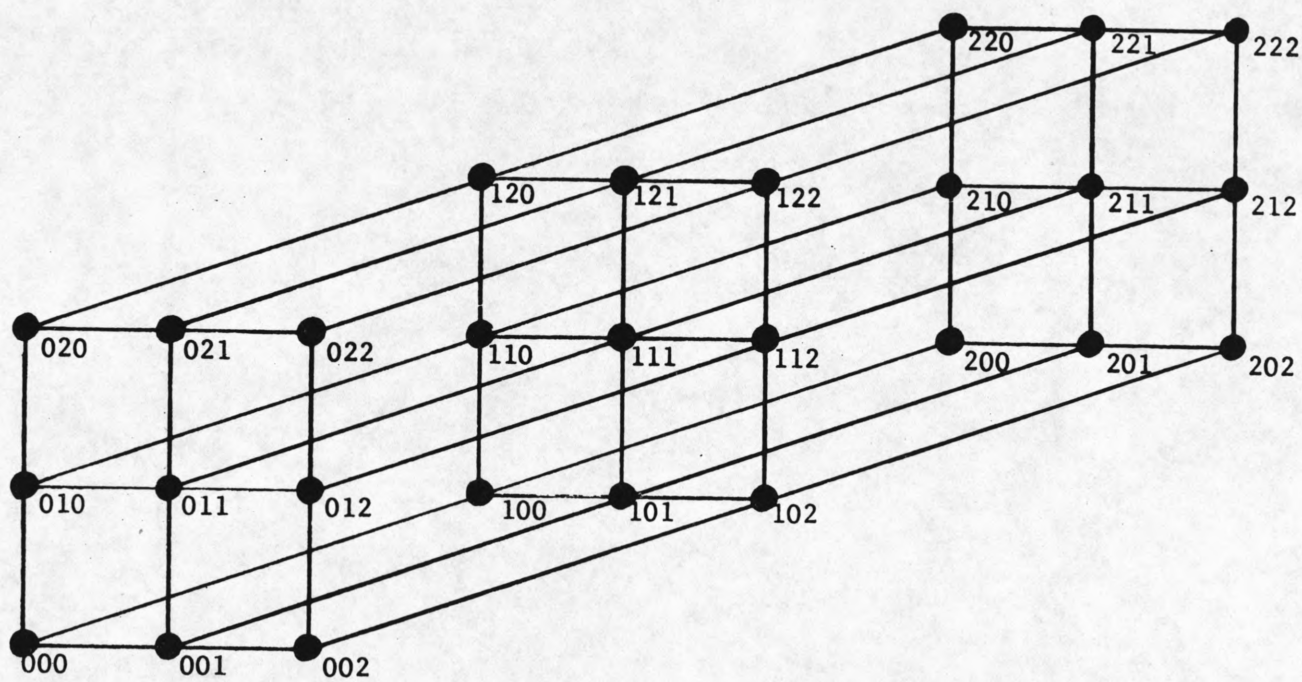

FIGURE 2

THE POST ALGEBRA $[P(3)]^3$

$y_1y_2y_3$	$C_0(y_1y_2y_3)$	$C_1(y_1y_2y_3)$	$C_2(y_1y_2y_3)$
000	222	000	000
001	220	002	000
002	220	000	002
010	202	020	000
011	200	022	000
012	200	020	002
020	202	000	020
021	200	002	020
022	200	000	022
100	022	200	000
101	020	202	000
102	020	200	002
110	002	220	000
111	000	222	000
112	000	220	002
120	002	200	020
121	000	202	020
122	000	200	022
200	022	000	200
201	020	002	200
202	020	000	202
210	002	020	200
211	000	022	200
212	000	020	202
220	002	000	220
221	000	002	220
222	000	000	222

TABLE 2

THE FUNCTION VALUES $C_i(y)$, $i = 0,1,2$, FOR $[P(3)]^3$

one-variable functions defined on the chain P(n) [8]. Thus it is possible to represent functions g(x) in G[P(n):x] by n-tuples, each of whose components is an integer i satisfying $0 \leq i \leq n-1$.

We turn now to the concept of equivalence for elements of G[P(n):x]. Based on [8,9] we let $\Phi_1(x),\ldots,\Phi_n(x)$ denote the n coordinate functions defined on P(n); that is, each $\Phi_i(x)$ is the n-tuple whose ith component has value n-1 and whose other n-1 components have value 0. Then it can be shown that every function g(x) in G[P(n):x] can be expressed uniquely as $g(x) = \sum_{k=1}^{n} g_k \cdot \Phi_k(x)$, where $0 \leq g_k \leq n-1$. This merely says that any one-variable function on P(n) can be expressed as a weighted sum of the coordinate functions. Using these coordinate functions, we now define the relation of equivalence for elements of the Post algebra G[P(n):x].

<u>DEFINITION 2</u>: Let $g(x) = \sum_{k=1}^{n} g_k \cdot \Phi_k(x)$ and $g'(x) = \sum_{k=1}^{n} g_k' \cdot \Phi_k(x)$ be elements of G[P(n):x]. Then g(x) and g'(x) are said to be equivalent, denoted by $g(x) \sim g'(x)$, if and only if $g_i = 0 \Leftrightarrow g_i' = 0$ for each $i = 1,\ldots,n$.

This definition states that two Post functions are equivalent if and only if corresponding coefficients in their weighted sums are either both zero or both nonzero. It is tedious, but not difficult, to verify that the relation of equivalence for Post algebras G[P(n):x] is an equivalence relation [8].

As an example of equivalent elements in a Post algebra G[P(n):x], consider the members g(x) = 021 and g'(x) = 012 of the lattice G[P(3):x] of Figure 2. The coordinate functions are $\Phi_1(x) = 200$,

$\Phi_2(x) = 020$ and $\Phi_3(x) = 002$. Then $g(x) = 021 = 0\cdot\Phi_1(x) + 2\cdot\Phi_2(x) + 1\cdot\Phi_3(x)$ and $g'(x) = 012 = 0\cdot\Phi_1(x) + 1\cdot\Phi_2(x) + 2\cdot\Phi_3(x)$. Since $g_1 = g_1' = 0$, $g_2, g_2' \neq 0$ and $g_3, g_3' \neq 0$, it follows from Definition 2 that $g(x) \sim g'(x)$. Now consider the elements $g(x) = 021$ and $g''(x) = 121$. Then $g(x) = 0\cdot\Phi_1(x) + 2\cdot\Phi_2(x) + 1\cdot\Phi_3(x)$ and $g''(x) = 1\cdot\Phi_1(x) + 2\cdot\Phi_2(x) + 1\cdot\Phi_3(x)$. Since $g_1 = 0$ and $g_1'' \neq 0$, it follows by Definition 2 that $g(x) \not\sim g''(x)$. Employing the isomorphism $G[P(n):x] \cong [P(n)]^n$, the relationships $g(x) \sim g'(x)$ and $g(x) \not\sim g''(x)$ are even more obvious. The elements 021 and 012 have zeros for precisely the same components, while the first components of 021 and 121 are neither both zero nor both nonzero.

We now introduce a special subset of the Post algebra $[P(n)]^n$. Define $[R(n)]^n$ to be the set of all elements $y = y_1 \cdots y_n$ in $[P(n)]^n$ having the property that $y_i \neq 0$ for all $i = 1,\ldots,n$. As an example of the set $[R(n)]^n$, consider $[R(3)]^3 = \{111, 112, 121, 122, 211, 212, 221, 222\}$. This set plays an important role in a necessary and sufficient condition, which we give in the next section, for two elements of $[P(n)]^n$ to be equivalent.

III. CHARACTERIZATIONS OF EQUIVALENCE

The first characterization of the relation of equivalence for one-variable Post functions employs a coset-like product involving $[R(n)]^n$. For $y \in [P(n)]^n$ we define $y\cdot[R(n)]^n$ to be the set of all products $y\cdot r$, where $r \in [R(n)]^n$. Since the multiplication operation of $[P(n)]^n$ is the componentwise minimum function [2], it is commutative. Thus it makes no difference in the sequel whether we use the left products $y\cdot[R(n)]^n$ or the (identical) right products $[R(n)]^n\cdot y$. With this in mind we adopt the convention of always using the left products.

<u>THEOREM 3</u>: If $y,z\in[P(n)]^n \cong G[P(n):x]$, then $y \sim z$ if and only if there exists an element w in $[P(n)]^n$ such that $y,z\in w\cdot[R(n)]^n$.

<u>PROOF</u>: Assume $y \sim z$. Then if $y = \sum_{k=1}^{n} y_k\cdot\Phi_k(x) = (y_1,\ldots,y_n)$, $z = \sum_{k=1}^{n} z_k\cdot\Phi_k(x) = (z_1,\ldots,z_n)$, where $y_i = 0$ if and only if $z_i = 0$ for each i. We claim that the element w of the theorem can be taken to be $y + z$. It must be shown that there exist elements $r,s\in[R(n)]^n$ such that $y = (y+z)\cdot r$ and $z = (y+z)\cdot s$. Define $r = r_1\ldots r_n$ by taking r_i to be some integer ℓ_i satisfying $0 < \ell_i \leq n\text{-}1$ if $y_i = 0$, and choosing $r_i = y_i$ if $y_i \neq 0$. Now if $y_i = 0$, then it follows from the coordinate function representations of y and z that $z_i = 0$. Therefore $((y+z)\cdot r)_i = (y_i+z_i)\cdot\ell_i = 0\cdot\ell_i = 0 = y_i$. On the other hand, if $y_i \neq 0$, it follows that $z_i \neq 0$. Thus $((y+z)\cdot r)_i = (y_i+z_i)\cdot y_i = y_i$. So in either case, $((y+z)\cdot r)_i = y_i$. Since this holds for all $i = 1,\ldots,n$, it follows that $y = (y+z)\cdot r$. As $r_i \neq 0$ for all $i = 1,\ldots,n$, we have $r\in[R(n)]^n$. Consequently, $y\in(y+z)\cdot[R(n)]^n$. Similarly, there exists $s\in[R(n)]^n$ such that $z = (y+z)\cdot s$. Thus $y+z$ can serve as the desired element w.

For the converse inclusion, assume there exists an element $w\in[R(n)]^n$ such that $y,z\in w\cdot[R(n)]^n$. Then there are elements r and s in $[R(n)]^n$ such that $y = w\cdot r$ and $z = w\cdot s$. Let $y = \sum_{k=1}^{n} y_k\cdot\Phi_k(x) = (y_1,\ldots,y_n)$. We show $z = \sum_{k=1}^{n} z_k\cdot\Phi_k(x) = (z_1,\ldots,z_n)$ satisfies

$z_i = 0 \iff y_i = 0$. Now if $y_i \neq 0$, $(w \cdot r)_i = w_i \cdot r_i \neq 0$. Therefore $w_i \neq 0$. Since $s_i \neq 0$, it follows that $w_i \cdot s_i = (w \cdot s)_i = z_i \neq 0$. On the other hand, if $y_i = 0$, then $(w \cdot r)_i = w_i \cdot r_i = 0$. As $r_i \neq 0$, we must have $w_i = 0$. Hence, $w_i \cdot s_i = (w \cdot s)_i = z_i = 0$. Therefore y and z are nonzero combinations of the same coordinate functions; that is, $y \sim z$.

As an example of this theorem, consider the elements $g(x) = 021$ and $g'(x) = 012$ of $G[P(3){:}x]$. It has already been shown that $021 \sim 012$ by using Definition 2. Now $g(x) + g'(x) = 022$ and $022 \cdot [R(3)]^3 =$ $\{022 \cdot 111, 022 \cdot 112, 022 \cdot 121, 022 \cdot 122, 022 \cdot 211, 022 \cdot 212, 022 \cdot 221, 022 \cdot 222\}$ $= \{011, 012, 021, 022\}$. Thus both 021 and 012 are members of $(g(x) + g'(x)) \cdot [R(3)]^3$. Conversely, any two elements of the class $022 \cdot [R(3)]^3$ are equivalent. This is easily verified by inspection, since for all elements of this class, $g_1 = 0$, $g_2 \neq 0$ and $g_3 \neq 0$.

It is interesting to note that not only are all the elements of the class $022 \cdot [R(3)]^3$ equivalent, but that there are no other members of $[P(3)]^3$ which are equivalent to any element of this class. Now consider the product $021 \cdot [R(3)]^3 = \{011, 021\}$. By Theorem 3, $011 \sim 021$. However, 012, which is not a member of $021 \cdot [R(3)]^3$, is also equivalent to 021, as has been shown. Thus $021 \cdot [R(3)]^3$ does not include all the elements equivalent to its members. It will soon be seen that what distinguishes $022 \cdot [P(3)]^3$ from $021 \cdot [P(3)]^3$ is that 022 is the least upper bound of the set of equivalent elements 011, 012, 021, 022. Since each Post algebra $[P(n)]^n$ is finite (it contains n^n elements), each class $y \cdot [R(n)]^n$ is also finite and, hence, has a least upper bound. Our next theorem characterizes the class of all elements of $[P(n)]^n$ equivalent to a given element of this lattice in terms of the left product of $[R(n)]^n$ by the least upper bound of this class. Before this characterization can be given, however, several preliminary results are needed. The first of these establishes that the least upper bound of the set of all elements equivalent to a given element y of $[P(n)]^n$ is also equivalent to y. As a notational convenience in the sequel, let $[y]$ denote the collection of all elements of $[P(n)]^n$ which are equivalent to y.

<u>LEMMA 4</u>: Let $y \in G[P(n):x]$ and $[y] = \{\alpha_1, \ldots, \alpha_m\}$. Then if $\ell = \mathrm{lub}\{\alpha_1, \ldots, \alpha_m\}$, $\ell \sim y$.

<u>PROOF</u>: If $y = \sum_{k=1}^{n} y_k \cdot \Phi_k(x)$, where each y_k satisfies $0 \leq y_k \leq n-1$, then for all $j = 1, \ldots, m$, $\alpha_j = \sum_{k=1}^{n} \alpha_{j_k} \cdot \Phi_k(x)$, where each α_{j_k} satisfies $0 \leq \alpha_{j_k} \leq n-1$. If $y_i = 0$, then $\alpha_{j_i} = 0$ for all $j = 1, \ldots, m$. Thus $\alpha_{1_i} + \ldots + \alpha_{m_i} = \ell_i = 0$. On the other hand, if $y_i \neq 0$, then each $\alpha_{j_i} \neq 0$. Hence, $\ell_i \neq 0$. Therefore ℓ can be written as a nonzero combination of the same set of coordinate functions as y; that is $\ell \sim y$.

One additional preliminary result is necessary before we can establish the next theorem.

<u>LEMMA 5</u>: Let $y, z \in [P(n)]^n$. If $y \sim z$ and $y \leq z$, then $y \cdot [R(n)]^n \subseteq z \cdot [R(n)]^n$.

PROOF: Let $w \in y \cdot [R(n)]^n$. Since $y = y \cdot U$ and $U \in [R(n)]^n$, it follows that $y \in y \cdot [R(n)]^n$. By Theorem 3, $y \sim w$. Since the relation of equivalence for elements of $[P(n)]^n$ is an equivalence relation, then $w \sim z$. As $w \in y \cdot [R(n)]^n$, there exists an $r \in [R(n)]^n$ such that $w = y \cdot r$. Then $w \leq y$. Since $y \leq z$, we have $w \leq z$. Therefore $w + z = z$. Using the technique employed in the proof of Theorem 3, we have that $w \in (z + w) \cdot [R(n)]^n$. Now $z + w = z$ implies $(z + w) \cdot [R(n)]^n = z \cdot [R(n)]^n$. Hence, $w \in z \cdot [R(n)]^n$; that is, $y \cdot [R(n)]^n \subseteq z \cdot [R(n)]^n$.

Employing Lemmas 4 and 5, we can now characterize the equivalence classes $[y]$ in terms of left products of $[R(n)]^n$ by the least upper bounds of these classes.

THEOREM 6: Let $y \in [P(n)]^n$ and let ℓ be the least upper bound of the set of all elements of this algebra which are equivalent to y. Then $[y] = \ell \cdot [R(n)]^n$.

PROOF: If $z \in [y]$, then $z \sim y$. By Lemma 4, $\ell \sim y$. Therefore $z \sim \ell$. By definition, $z \leq \ell$. Hence, $z \cdot [R(n)]^n \subseteq \ell \cdot [R(n)]^n$ by Lemma 5. Since $z \in z \cdot [R(n)]^n$, $z \in \ell \cdot [R(n)]^n$; that is, $[y] \subseteq \ell \cdot [R(n)]^n$.

For the converse inclusion, let $z \in \ell \cdot [R(n)]^n$. As $\ell \in \ell \cdot [R(n)]^n$, it follows from Theorem 3 that $z \sim \ell$. Since $\ell \sim y$ by Lemma 4, we have $y \sim z$. Thus $z \in [y]$; that is, $\ell \cdot [R(n)]^n \subseteq [y]$.

For an example of Theorem 6, consider the element 201 of $[P(3)]^3$. The members of this algebra which are equivalent to 201 are 101, 102, 201 and 202. The least upper bound of this set is 202. Thus $\ell \cdot [R(3)]^3 =$

$\{202 \cdot 111, 202 \cdot 112, 202 \cdot 121, 202 \cdot 122, 202 \cdot 211, 202 \cdot 212, 202 \cdot 221, 202 \cdot 222\} = \{101, 102, 201, 202\}$, as guaranteed by the theorem.

If we now examine the left products $101 \cdot [R(3)]^3$, $102 \cdot [R(3)]^3$ and $201 \cdot [R(3)]^3$, we find that all of these classes are proper subsets of $[201]$. For $101 \cdot [R(3)]^3 = \{101\}$, $102 \cdot [R(3)]^3 = \{101, 102\}$ and $201 \cdot [R(3)]^3 = \{101, 201\}$. It will now be shown that this is true in general in Post algebras $[P(n)]^n$.

<u>THEOREM 7</u>: Let $y, z \in [P(n)]^n$ and let $\ell = \text{lub}\{w \in [P(n)]^n | w \sim y\}$. If $z \in [y]$ and $z < \ell$, then $z \cdot R \subset [y]$.

<u>PROOF</u>: By Lemma 5, $z \in [y]$ and $z < \ell$ imply $z \cdot [R(n)]^n \subseteq \ell \cdot [R(n)]^n$. Since U is the largest element of $[R(n)]^n$, it follows that $z \cdot U = z$ is the largest member of $z \cdot [R(n)]^n$. Thus if $z < \ell$, then $\ell \notin z \cdot [R(n)]^n$. Therefore $z \cdot [R(n)]^n \subset \ell \cdot [R(n)]^n$. Consequently, $z \cdot [R(n)]^n \subset [y]$ by Theorem 6.

Combining Theorems 6 and 7, we obtain a necessary and sufficient condition on z for the equivalence class $[y]$ to be equal to the set $z \cdot [R(n)]^n$.

<u>THEOREM 8</u>: If $y, z \in [P(n)]^n$ and $z \sim y$, then $[y] = z \cdot [R(n)]^n$ if and only if z is the least upper bound of the class $[y]$.

This completes our work on the equivalence classes $[y]$. It is the authors' hope that these first results on the relation of equivalence for one-variable functions defined on a Post chain will stimulate investigation in this area leading to a comprehensive theory of multivalued switching systems.

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13. ABSTRACT

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